

Pseudolites

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I. Introduction

THE Global Positioning System (GPS) has reached operational status and is now being used internationally as a means for accurate, world-wide, all-weather positioning and navigation based on pseudonoise signals transmitted by a constellation of 24 satellites. Pseudolites (PLs) are ground-based transmitters that can be configured to emit GPS-like signals for enhancing the GPS by providing increased accuracy, integrity, and availability.^{11,21}

Accuracy improvement can occur because of better local geometry, as measured by a lower vertical dilution of precision (VDOP), which is important in aircraft precision approach and landing applications. Accuracy and integrity enhancement can also be achieved by employing a PL's integral data link to support differential (DGPS) modes of operation and timely transmittal of integrity warning information. Availability is increased because a PL provides an additional ranging source to augment the GPS constellation.

Although the use of PLs offers many potentially significant benefits, a number of technical issues must also be addressed. One is the PL signal power level and the associated "near-far" problem that a user receiver may experience, depending upon the dynamic range of signal strength encountered as the distance to a PL changes. Other issues include deployment requirements, signal data rate, signal integrity monitoring, and user antenna location and sensitivity. This chapter addresses PLs from the perspectives of signal design considerations, integrated DGPS/PL considerations, and testing activities for assessing the reality and mitigation of various technical issues associated with using PLs.

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II. Pseudolite Signal Design Considerations

Pseudolites operating within the GPS frequency bands (L_1 : 1565–1585 MHz or L_2 : 1217–1237 MHz) can be configured to serve a limited area with a power level low enough to preclude appreciable interference to standard GPS signals. One application is the PL bubble concept proposed for operating near the ends of airport runways to augment real-time kinematic positioning based on GPS carrier-phase measurements³ (also see Chapter 15, this volume).

For PLs designed to cover a larger area, such as an entire airport or terminal area, potential interference to GPS signals is a key technical issue. Although operation outside the GPS bands is a possibility, this option ultimately adds rf front end complexity and cost to user equipment.

In this chapter, a PL signal structure is described that can operate within the L_1 band and mitigate or virtually eliminate the near–far issue. First, however, a brief review is given of previous implementations used on DOD test ranges and other approaches proposed for civil applications.

A. Previous Pseudolite Designs

Table 1 compares two DOD implementations and three other proposed approaches in terms of the diversity of techniques employed for interference mitigation: signal (code, frequency, time) and spatial separation. Pseudolites, called ground transmitters (GTs) during the Phase I GPS program, were used to augment GPS for testing user equipment at Yuma, AZ before there were enough satellites for navigation.⁴ The PL signal structure was the same as used on the GPS satellites, except for the data content. In those days, interference to the satellite signals was avoided by using different gold codes (Code Division Multiple Access, or CDMA) and keeping the user equipment under test at an adequate distance from the four GTs to prevent dynamic range sensitivity problems. The same was true in more recent Space Defense Initiative (SDI) testing with GTs built by Stanford Telecom (STel) to augment GPS for DOD's Range Applications Program. However, a Time Division Multiple Access (TDMA) signal (on–off scheme) was also implemented to prevent interfering with a co-located GPS receiver that provided real-time synchronization to the GPS satellites. At transmit times, this receiver simply blanked out the signal.

The use of PLs for civil aviation applications was first proposed in 1984.⁵ The “near–far” problem was recognized and addressed at that time, along with suggested solutions, but not carried any further. Three signal diversity techniques suggested as a solution were as follows:

- 1) Pulsed signals with random or fixed cycle rates, a TDMA variation
- 2) Signals transmitted at a frequency offset from L_1 (1575.42 MHz), but within the same frequency band as GPS, a variation of frequency division multiple access (FDMA)
- 3) Alternative codes that have a longer sequence than the existing GPS codes, a variation of CDMA

The first two alternatives were favored, with a preference toward the first. Although technically viable, the FDMA approach, including the possible use of L_2 (1227.6 MHz) or frequencies outside the GPS band, was considered more costly given the state of GPS receiver technology at the time. The CDMA

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Table 1 Pseudolite interference mitigation techniques—previous designs

Pseudolite signal implementation (I) or recommendation (R)	Interference mitigation techniques			
	Code division multiple access	Frequency division multiple access	Time division multiple access	Pseudolite spatial separation
(I) Yuma ground transmitters for the Phase I GPS tests (1977)	Different gold codes	None	None	Unspecified
(I) Stanford Telecom ground transmitters for Space Defense Initiative (1989)	Different gold codes	None	Pulsed (simple on-off)	Unspecified
(R) Reference 5	Longer sequence codes	L_2 or $L_1 \pm 15$ MHz	Pulsed at random or fixed rate	Unspecified
(R) Reference 6	Different gold codes	None	Pulsed with: 10% duty cycle, random pattern	≥ 30 km
(R) Reference 7	Different gold codes	$L_1 \pm \Delta f$ ($\Delta f \geq 30$ kHz)	Pulsed with: 1:11 duty cycle, fixed pattern, fixed offset between PLs	No constraint

approach with different pseudorandom noise (PRN) codes could be part of the diversity solution, but longer sequence codes would not add significant margin against cross-correlation interference.

As part of the Radio Technical Commission for Maritime (RTCM) user activities, a more definitive pseudolite signal structure was proposed in 1986.⁶ All three of the above multiple access techniques were considered, but pulsed TDMA was the only approach recommended, because it made the least impact on the design of GPS receivers based on the state of technology at the time. Subsequently, flaws in that TDMA scheme were observed with respect to a class of “nonparticipating” receivers, some of which are still in use today. This led to a modified TDMA scheme, which was proposed in 1990.⁷ Despite these proposals, fear of the near-far problem remained, and rightly so, because a limited interference margin can still exist with only code (CDMA) and pulsing (TDMA) employed.

B. New Pseudolite Signal Design

Fortunately, GPS receiver technology has advanced to a point that the FDMA approach is now viable, which improves the solution to the near-far interference

issue significantly. As a result, a more effective signal structure has been proposed that combines: good C/A codes a frequency offset that takes advantage of the code cross-correlation properties, and a good pulsing scheme.⁸

1. C/A Codes

A recent study identified 19 C/A codes that were considered best (in terms of cross-correlation level) of the 1023 possible codes in the GPS C/A-code family.⁹ These codes were selected for use with the FAA's wide area augmentation system (WAAS), which will augment GPS with a ranging signal transmitted at L_1 from geostationary orbit. Although the 19 codes selected were the best, there remain 712 balanced codes, almost as good, that could be used for PLs. (See also Chapter 3 in the companion volume for additional discussion of Gold codes and cross-correlation properties.)

2. Frequency Offset

The proposed offset is 1.023 MHz on either side of L_1 at 1575.42 MHz, which places the PL carrier in the first null of the GPS satellite C/A-code spectrum. This offset is much more effective and one that at least some current GPS receivers can accommodate, because it resembles a large Doppler frequency offset. Furthermore, it virtually eliminates any code cross-correlation with the GPS satellite signals. This spectrum is shown in Fig. 1, which is a typical spectrum of a GPS C/A code [PRN 2]. It was reported years ago that the cross-correlation between C/A codes at different frequencies was simply the magnitude of another C/A-code spectral line at the offset frequency.¹⁰ This property has since been verified. Because all the codes have similar spectral characteristics, the spectrum of any C/A code tells the story. As indicated in Fig. 1 and discussed further in Appendix A, the spectral lines near the null are well below -80 dB and are 60 dB lower than those at a zero frequency offset. This property has been used to

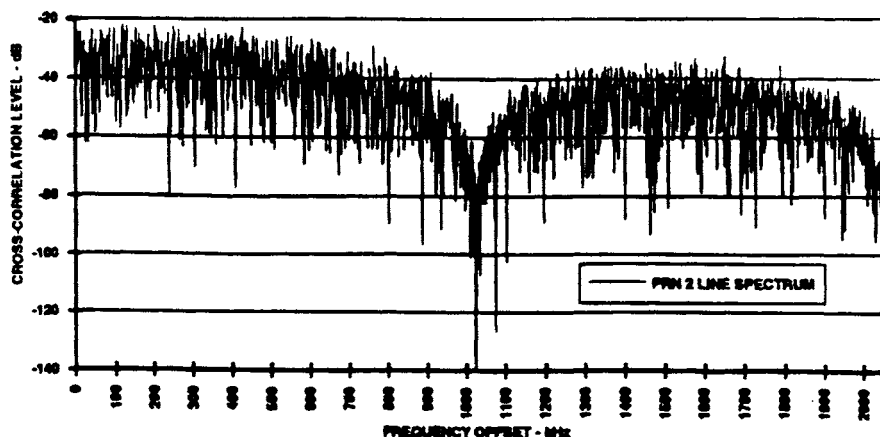


Fig. 1 Spectrum of the GPS C/A-code PRN 2.

advantage in the Global Orbiting Navigation Satellite System (GLONASS) signal structure.¹¹ All the GLONASS signals have the same maximum length codes with the first nulls at 511 kHz offsets from the carrier. The GLONASS FDMA signals are spaced at 562.5 kHz intervals, 51.5 kHz from the nulls of the adjacent signal.

Recently, it has been shown that cross-correlation levels can be somewhat higher than indicated above when the code chip boundaries are not aligned.¹² However, when the carrier frequency is offset by 1.023 MHz, and the carrier/code frequency ratio of 1540 is maintained, the code of the PL is shifting with respect to the satellite codes at a rate of over 664 chips per second. Thus, any cross-correlation is noise-like and averages to a lower rms level. As pointed out, this is still interference.¹² An in-band frequency offset of 1.023 MHz lowers the rms interference by about 8 dB, but does not eliminate it. However, it does eliminate cross-correlation problems.

3. Pulsing Scheme

The FDMA approach described above is a variation of an earlier proposal, which suggested a continuous signal with a longer code on the fringe of the allocated L_1 band at 1560 or 1590 MHz. However, a *continuous* in-band PL signal could still cause interference problems, depending upon its power level compared to GPS satellite signals. For example, a PL signal whose power is set to be received at 37 km (20 n.mi.) will be 60 dB stronger at 37 m (0.02 n.mi.), which will be approximately 30–40 dB above the noise. Thus, although offsetting the frequency is a good idea, TDMA pulsing may still be required to avoid this situation. Furthermore, if TDMA is used, there is no reason to operate on the fringe of the L_1 allocated band, because that large a frequency offset and a longer code are both undesirable due to the additional burden on the GPS receiver, even with modern technology.

With a good pulsing scheme, the impact on the reception of GPS signals can be made essentially transparent. The receiver treats it as a continuous signal, provided that it is designed to suppress pulse interference, as most modern receivers are, even if by accident. As noted in the RTCM work,⁶ any hard-limiting receiver or “soft-limiting” receiver will clip the pulses and limit their effect, but still pass more than enough pulse power to track the PL signal itself. A “soft-limiting” receiver clips the incoming signal-plus-noise at two to three times the rms noise level, resulting in more, but still negligible degradation in signal-to-noise performance.

All modern digital receivers are either hard-limiting or possess the soft-limiting property through precorrelation quantization. Although this is not true for the older, analog military receivers, their wideband automatic gain control (AGC) suppresses the pulses for the same effect. Any cross-correlation problems in those receivers caused by PL transmissions are eliminated with the proposed frequency offset.

The pulsing scheme presented here differs from previous proposals.^{6,7} It accounts for the fact that the PL message symbol rate could be 50 N sps (where $N = 1, 2, 4, 5, 10$, or 20). The recommended pulse pattern is illustrated by the Xs shown in Fig. 2. These repeat every 11 ms, and thus, would never be synchro-

Slot No.	C/A Code Cycles																						
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23
1	X				Y							X				Y							X
2		X				Y							X				Y						X
3			X				Y							X				Y					
4				X				Y							X				Y				
5					X				Y							X				Y			
6						X				Y							X				Y		
7							X				Y							X				Y	
8	Y							X				Y							X				Y
9		Y							X				Y							X			Y
10			Y							X				Y							X		
11				Y							X				Y							X	

X = PL₁ Pulses, Y = PL₂ Pulses, etc.

Fig. 2 Illustration of pseudolite pulse pattern.

nous with a received GPS bit pattern. Each code cycle (1 ms) is divided into 11 slots, each with a width of $1/11,000$ s ($90.90909 \mu\text{s}$). The pulses would never be synchronous with a received GPS bit pattern, as would occur with the RTCM design.⁶

Only one of these slots contains a pulse, so the duty cycle is $1/11$. There will be $20/N$ pulses per symbol, but only one is required because the receiver would integrate the energy over the entire symbol period. Because every slot is filled once every 11 ms, the entire C/A code would be received every 11 ms. The clocking rate for the slots is $1/93$ of the C/A-code chipping rate of 1.023 MHz, or 11 kHz. It is noted in Appendix B that with a duty cycle of $1/11$, the loss of GPS satellite signal-to-noise ratio, in either an analog receiver with pulse suppression, or a digital receiver with natural *soft-limiting*, is less than 1.5 dB when close to the PL.

Potential mutual interference when multiple PLs are installed in an area was reported in the RTCM work.⁶ With the RTCM pulse pattern, a minimum distance between PLs would be required, because the transmission of pulses simultaneously from each PL could result in the simultaneous reception of multiple pulses. But, that need not be, because the pulse timing of multiple PLs can be offset. Unfortunately, the RTCM pattern had irregular times between pulses, so that there would still be some simultaneous receptions.

However, with the pulse pattern shown in Fig. 2, a suitable pulse-timing offset would prevent simultaneous reception from ever occurring, except at large distances where the PL signal would be of little consequence. For example, consider two PLs where the pulse timing was offset by 4 ms as indicated by the Xs and Ys in Fig. 2. Because the minimum transmit time separation between X and Y pulses is $4/11$ ms, only receivers with a differential range more than 110 km from the PLs would encounter simultaneous reception. At that distance, the received powers of either PL would be negligible.

4. Pseudolite Carrier Tracking

A potential misconception about pulsing a PL signal is that it will prevent the tracking of PL carrier phase. This is not true if the pulses occur at a high enough rate, which is the case in the proposed scheme. One or more pulses will always be integrated along with noise over each symbol. The result is transparent to the tracking loops, because the phase change due to Doppler uncertainty over a symbol is negligible. To the tracking loops, it looks like a continuous signal.

5. Pseudolite Transmit Power

Given an average received power P_r through a receiving antenna (with gain G_a) at a distance d (in n.mi.) from a PL, the average transmitted power (P_T) at $L_1 \pm \Delta f$ is as follows:

$$P_T \approx P_r + 20 \log_{10} \left(\frac{4\pi d}{\lambda_1} \right) - G_a$$

$$= P_r + 20 \log_{10} d + 101.75 - G_a$$

where $\lambda_1 = 0.00010275$ n.mi. is the signal wavelength corresponding to L_1 . As an example, consider an average received power of -130 dBm at 20 n.mi. (37 km) through an antenna gain of -10 dB (assumed for a small negative elevation angle). The average transmitted power is 7.77 dBm, or about 6 mW. The peak power for a duty cycle of $1/11$ is then 18.18 dBm, or about 66 mW.

III. Integrated Differential GPS/Pseudolite Considerations

The introduction of PLs has two key objectives: signal augmentation and data link enhancement. The first is to increase the number of available signals and, thereby, improve or maintain the geometrical quality for user position determination. The second is to provide an integrated capacity for supplying key data to users for GPS (and PL) integrity warning and differential corrections to improve positioning accuracy via code-based local differential GPS (LDGPS) and potentially carrier phase-based kinematic differential GPS (KDGPS) techniques.

Although the definition of a compatible signal format is one of the most critical requirements for meeting these objectives, there are other implementation-related aspects that need consideration as well. This section addresses five of these: PL siting, PL time synchronization, antenna location on a user aircraft, the PL data message, and the filter algorithm for integrated GPS/PL measurement processing.

A. Pseudolite Siting

It is well known that the GPS geometry (quantified in terms of HDOP and VDOP factors) will vary over time and user location, even with a full 24-satellite constellation¹³ (also see Chapter 5, the companion volume). It is also known that at times significant degradation ($\text{VDOP} \gg 6$) can occur if fewer satellites are active. The utility of PLs for geometric enhancement lies in the fact that lower DOP values with less temporal variation can be achieved.

Figures 3 and 4 illustrate representative HDOP and VDOP profiles determined for a full constellation¹³ and with one satellite inactive for cases where the user employs the best four GPS satellites or “all-in-view” with augmentation by one or two PLs.² In this illustration, the user was assumed to be at an altitude of 200 ft (the decision height for a Cat. I approach) with the PLs located 1 n.mi. ahead and/or 2 n.mi. behind. It is evident that in this situation, a two-PL augmentation would enable $\text{HDOP} \leq 1.0$ and $\text{VDOP} \leq 1.5$ continuously. At those levels, code-based LDGPS with the capability to correct uncorrelated pseudorange errors to $\leq 1\text{m}$ (2σ) would meet the current Cat. III (horizontal) and Cat. II (vertical) sensor accuracy requirements of 4.1m and 1.4m (2σ), respectively for aircraft precision approaches.¹⁴ With the addition of ranging-capable satellites in geostationary orbit (e.g., via the FAA’s WAAS implementation) to offset one or two inactive GPS satellites an even lower VDOP is possible with favorable implications for meeting the most stringent Cat. III (vertical) accuracy requirement of 0.4m (2σ).¹⁴

An alternate approach to meeting the Cat. III vertical accuracy requirement for precision approaches is based on KDGPS techniques with PL augmentation. Recent flight tests have demonstrated that a varying user/PL geometry will permit rapid resolution of carrier-range ambiguities to provide KDGPS-based measurements with centimeter-level precision³ (also see Chapter 15, this volume). The configuration adopted for this technique utilizes a *bubble* concept in which two PLs located ahead of the runway threshold on each side of the glideslope transmit a standard, unpulsed L_1 -C/A signal at a low power level sufficient to be received within a hemispherical bubble centered at each PL. A descending aircraft would enter the common bubble zone, acquire the PL signals, resolve GPS carrier-phase ambiguities, and emerge to continue the approach with KDGPS-based corrections supplied via a separate (non-PL) datalink. As proposed, four active PLs per runway would be needed to support landings in either direction.

On the other hand, wider coverage *large bubble* PLs with interference mitigation inherent in the signal design could provide service to multiple runways over an entire airport region. Location of the PL antenna off runway and at suitable elevation would also be desirable from the standpoint of user antenna requirements (discussed in Sec. II.C). Pseudolite-siting requirements (number and location) to accommodate multirunway situations, user antenna considerations, and local constraints are key issues needing further analysis and testing.

B. Pseudolite Time Synchronization

Synchronizing a PL’s clock to GPS time can be achieved in two ways—one in which a PL is collocated with an LDGPS reference receiver (RR), and one in which it is remote from the RR that is tracking its transmitted signal. In the latter case, the RR sends corrections to the remote “slave” PL to correct itself and also sends message data (e.g., code- and/or carrier-based DGPS corrections) to be transmitted. These two different PL configurations are illustrated in Fig. 5. In the co-located configuration, the RR shares the transmit/receive antenna with the PL, which also allows for self-calibration.

The type of configuration used would depend upon whether or not there is more than one PL at a given local region. If there is only one, the collocated

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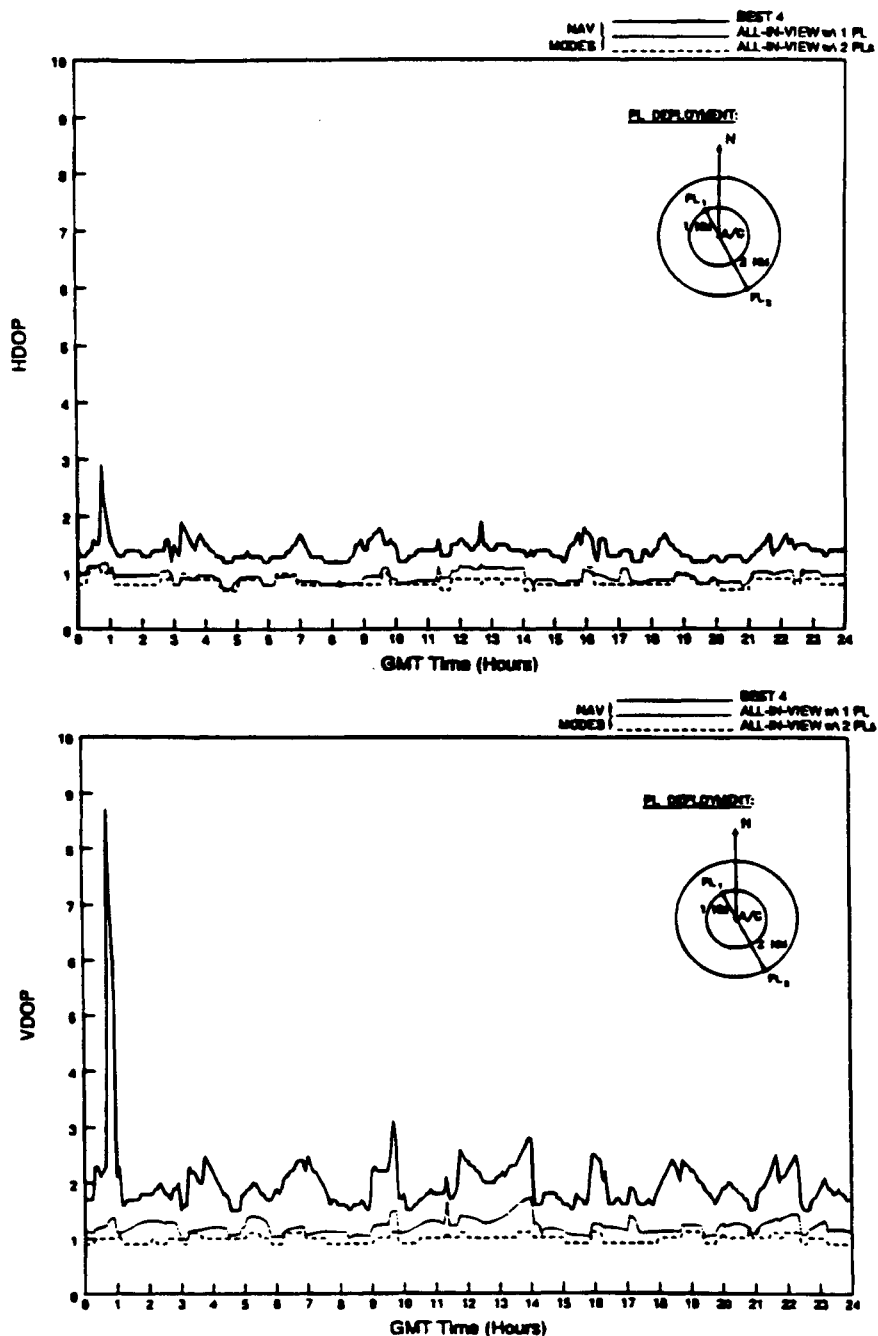


Fig. 3 Worst-case dilution-of-precision profiles at 35°N with and without PLs for a full (24) GPS constellation.

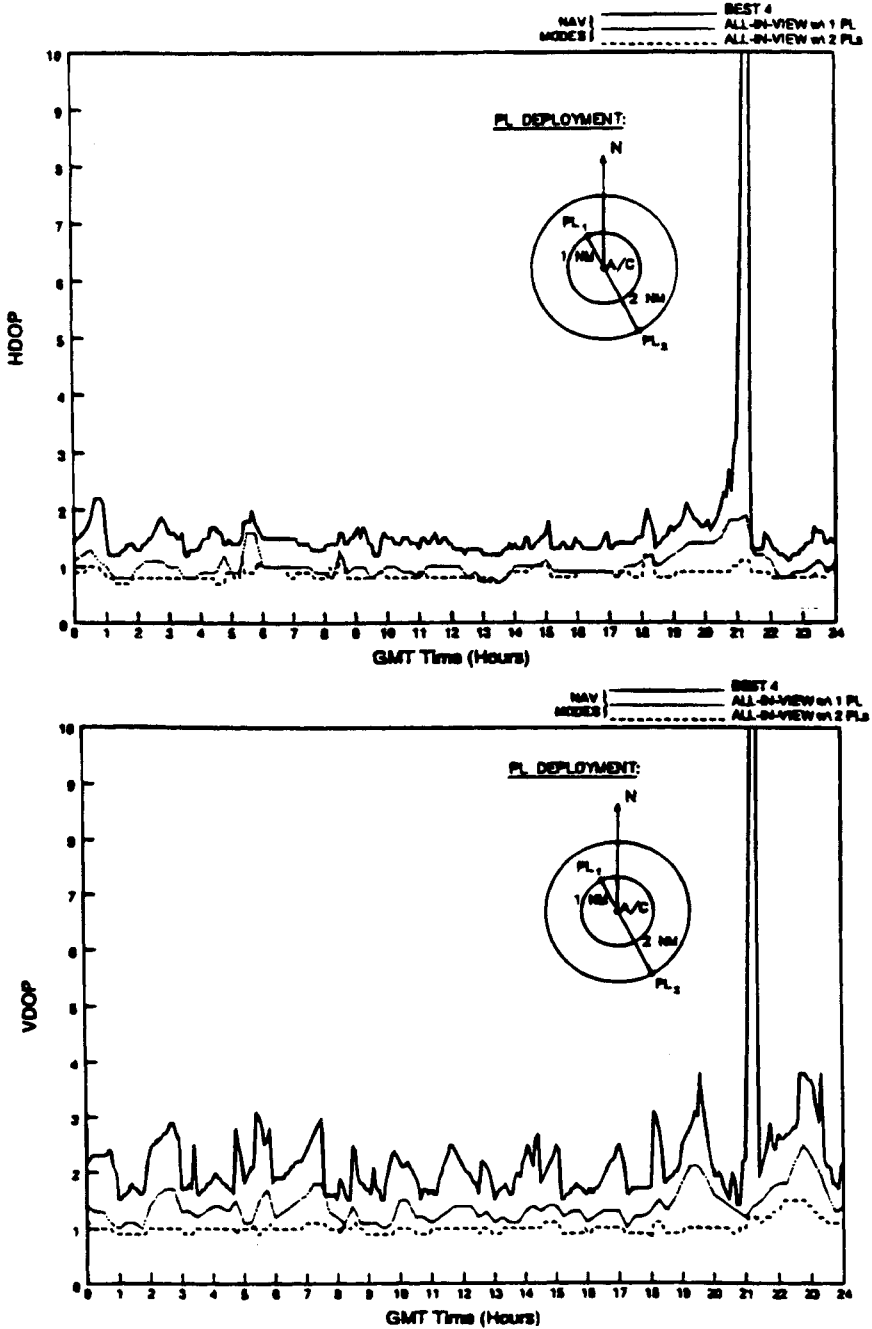
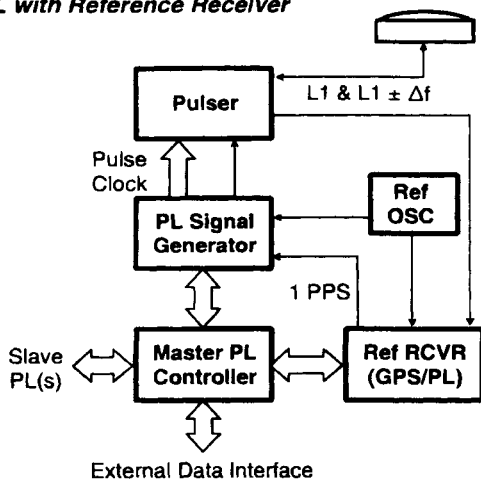


Fig. 4 Worst-case dilution-of-precision profiles at 35°N with and without PLs for a GPS constellation with one inactive satellite.

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a) Master PL with Reference Receiver



b) Slave PL

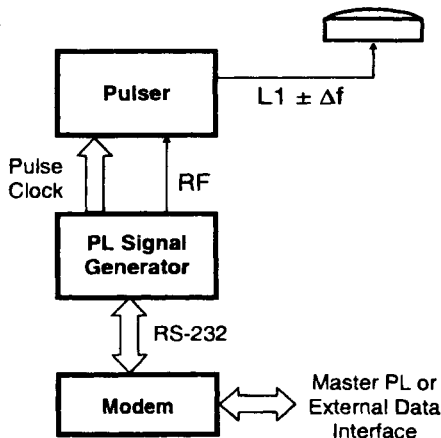


Fig. 5 Master and slave pseudolite configurations.

approach is more desirable, especially if line-of-sight (LOS) visibility to the RR might be a problem. If there is more than one PL, the remote approach with a common RR for synchronization is more desirable. However, if LOS visibility problems exist, having a RR collocated with each PL would enable time synchronization via GPS common-view, time-transfer techniques. However, only one RR can be used for deriving satellite LDGPS corrections. In this case, master and slave PLs would be designated with the master clock and LDGPS corrections derived centrally and distributed to each remote PL, which would slave its time to the master RR clock. This approach is similar to that used by various DOD

test and training ranges coordinated by the Range Applications Joint Program Office (RAJPO), for which P-code PLs were developed by Stanford Telecom.

1. Master Pseudolite Configuration

Both the RR and the PL signal generator derive their timing coherently from the same stable frequency standard. The signal generator/pulsar electronics module pulses the transmission of the PL signal in order to minimize interference to both participating and nonparticipating GPS receivers, as discussed in Sec. IB3. This multiplexing also allows the RR to receive the satellite signals via the same antenna. In fact, by providing a suitable calibration path, the RR can also track the output of the signal generator. In this way, the collocated PL is self-calibrating, and the transmitted PL signal will be synchronized to the same clock that is used to derive the differential GPS corrections. This is true even in the multiple-PL scenario, where the slave PLs receive differential corrections from the master PL. In this case, the time solutions of the slave PL receivers will be referenced to the master PL's clock, because the differential corrections are computed with respect to that clock.

2. Slave Pseudolite Configurations

The slave PLs need not have receivers if they can be tracked by the RR. The RR simply supplies corrections to the PL for correcting its clock via a number-controlled oscillator (NCO) and provides the satellite differential corrections for modulation on the PL signal. Because the RR can update the PL continuously, a good quality crystal oscillator will suffice as its frequency reference. Otherwise, its configuration is identical to the co-located configuration but without the RR and the self-calibration path.

C. User Aircraft Antenna Location

Reception of the PL signal by a user will be affected by the aircraft antenna location and the PL position relative to its approach path. Ideally, the PL signals would be received by a top-mounted antenna, the same one used for receiving the satellite signals. This could be accommodated by locating the PL antenna at an appropriate elevation and offset from the glideslope. Even if the line of sight to the PL is below the aircraft antenna horizon, the increased signal level in the vicinity of the PL will tend to cancel the loss in antenna gain.

If the aircraft were to pass directly over a PL, a larger angular gradient would be available to support the positioning process. However, a bottom-mounted antenna and a separate front end to interface the antenna(s) to the receiver may be needed. A configuration with a bottom-mounted antenna would likely have more sensitivity to ground-reflected multipath and interference. A top-mounted antenna is less sensitive, because reflective surfaces on the aircraft are typically small relative to the C/A-code chip width (293 m). In addition, the use of a bottom-mounted antenna would introduce additional sensitivity to lever arm uncertainty and knowledge of aircraft attitude information. On the other hand, it could be used directly, if a PL were to be employed only as a datalink. A bottom antenna is required for the small bubble concept.³ Obviously, further

analysis and testing are needed before the aircraft antenna location issue is resolved, one way or the other.

D. Pseudolite Signal Data Message

A PL offers the possibility for an order-of-magnitude increase in the data rate (up to 1000 bps vs 50 bps for GPS) that can be supported via a GPS-compatible signal with essentially a firmware change in the user receiver. Validation of data link performance at this rate is a key test objective. A closely related issue is the PL message capacity required to support GPS and PL integrity updates and DGPS corrections (code and/or carrier).

Some have proposed that for the latter, a 2400 bps data link would be needed to supply raw carrier-phase and other GPS information from a reference receiver at a 0.5–1.0 Hz update rate. Efforts by RTCM SC-104 (Special Committee 104) and others contend that KDGPS could be supported with much lower data transfer requirements (≤ 1 kbps) based on the use of carrier phase *corrections*, not raw data.¹⁵ A related issue for testing is whether PL aiding of carrier-phase ambiguity resolution and cycle slip maintenance procedures could reduce the data requirements still further.

1. General Format

For efficiency, the general format is patterned after the WAAS format¹⁶ with three differences—it may or may not include forward error correction (FEC), a 7-bit distributed time word is added and the data rate can be higher than 250 bps, up to 1 kbps without FEC. This general PL format is shown in Fig. 6. The time word is added as a convenience to the user receiver, because PL time is not the same as GPS time. This is because the code-chipping rate is offset in frequency by approximately ± 664 chips per second, the feature that eliminates cross-correlation with the GPS signals. The actual PL frequency can be chosen (at an offset from the spectral null of 710.4166667 Hz) so that the PL week is exactly 393 s shorter or longer than the GPS week. In fact, PL time is, at any GPS time, as follows:

$$t_{PL} = \left(1 \pm \frac{393}{604,800} \right) t_{GPS}$$

depending upon which null is selected. The PL time is distributed over three 250-bit subframes making up a 21-bit word. The 21-bit time word represents the

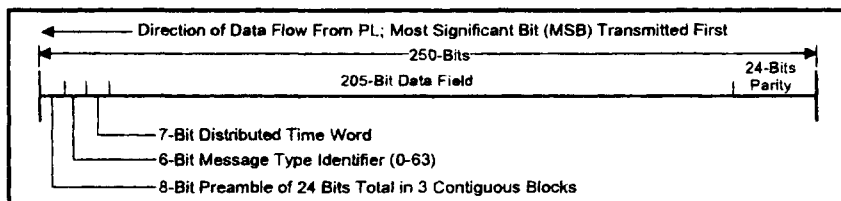


Fig. 6 Pseudolite message format.

PL time of week at the start of the currently transmitted 24-bit preamble, starting with 0 at the beginning of the week. This PL time word also serves as the reference time for the data in the messages.

The 24-bit parity is the same as the cyclic redundancy check (CRC) parity specified for the WAAS.¹⁶ The data field consists of 205 information-bearing bits.

2. Message Types

To avoid confusing the message types with those of the WAAS, the message type numbers start at 40. Table 2 lists a tentative set of PL message types. Every message would include a certain number of GPS/PL signal integrity flags to provide a short time to alarm capability.

3. Message Content

The contents of the messages are somewhat different from the message contents defined for the RTCA SCAT-I (Special Category I) DGPS.¹⁷ This is to accommodate the 205-bit data fields and to provide data that are more consistent with Category II and III precision approach requirements. The SCAT-I messages are quite inefficient in requiring too much signal bandwidth. Preliminary message contents show more than adequate bandwidth in using the maximum 1 kbps capability of the proposed PL signal. Every message broadcast contains integrity flags for several PRN numbers, with a minimum of 11, including the transmitting PL. This allows for a positive integrity indication at least once per 0.5 s. PRN numbers that accompany the flags do not have to be broadcast in order, so an alarm can always be inserted in any 0.25 s message providing a maximum time to alarm of 0.5 s.

Message Type 42 consists of a slight modification to the content of the corresponding RTCM carrier-phase corrections message.¹⁵ Each can accommodate four satellites/pseudolites. Therefore, both pseudorange and carrier-phase corrections can be broadcast.

E. GPS/Pseudolite Navigation Filter Considerations

When pseudorange (or carrier-range) measurements are processed by the user navigation filter, modeling to accommodate the nonlinearity in the measurement

Table 2 Pseudolite message types

Type	Contents
40	Don't use this PL for anything (PL testing)
41	Integrity flags/Pseudorange corrections
42	Integrity flags/Carrier-phase corrections (if required)
43	Integrity flags/PL location and PRN assignment
44	Integrity flags/PL almanacs
45	Integrity flags/Precision approach path definition
46	Integrity flags/Special message
63	Null message—Alternating 1s and 0s

model can be ignored in the case of satellites, but not necessarily for PLs. As the user range to the PL diminishes during an approach, the impact of the nonlinearity is to introduce an apparent bias into the measurements. If this error is comparable to the measurement error, then a standard extended Kalman filter (EKF) will yield inferior performance. Filter divergence may occur, as the combination of measurement and nonlinearity error exceeds the filter's own computation of rms error, and it rejects new measurement data.

One approach to preventing filter divergence of this sort is to choose sufficiently large a priori measurement variances that include worst-case nonlinearity effects. Unfortunately, this requires identification of what *is* worst case. More importantly, high constant measurement variances will cause sluggish performance at other times when there is a negligible nonlinear effect present.

Another approach, as outlined in Appendix C, is to increase the filter sophistication by using a Gaussian second-order (GSO) filter that is similar to a (linearized) EKF but includes a quadratic component of the measurement nonlinearity. The key benefit is that it offers improved performance when the nonlinear effect is present but reverts naturally to a standard EKF when it is not. A possible downside is that more software and processing time are required, although some approximations are possible depending upon the scenario. Pseudolite siting relative to the approach trajectory will also be a factor in this.

IV. Pseudolite Testing

The PL concept for augmenting GPS, as described above, is intended to provide users with the following potential benefits: 1) enhanced local area navigation performance via integrated GPS/PL positioning using single or multiple PL signals with airportwide or even terminal area coverage depending upon the power level and antenna configuration (s) employed; 2) more data link capacity to support DGPS operations directly (code and/or carrier-based) at a multiple N of 50 bps (where $N = 2, 4, 5, 10$, or 20); and 3) mitigation of potential interference to standard GPS signals through the code, frequency, and time diversity techniques employed in the PL signal design.

Initial testing to verify these features has been conducted under a research and development project sponsored by the FAA,¹⁸ and further comprehensive testing is planned. The following subsections describe the test results.

A. Pseudolite Interference Testing

Initial laboratory tests to evaluate PL interference impacts on GPS receivers was conducted by applying simulated GPS and PL signals at various power levels to several commercial receivers. After initial signal acquisition and steady-state operation by the GPS receiver was reached, a PL signal was injected at gradually increasing power levels. Signal quality reported in terms of GPS carrier power-to-noise power density ratio (C/N_0) was recorded as a function of average PL signal power. The plots in Fig. 7 show the test results obtained for two receivers under the following PL signal conditions: 1) no pulsing, no frequency offset; 2) no pulsing, frequency offset applied; 3) pulsing applied, no frequency offset; and 4) pulsing and frequency offset applied.

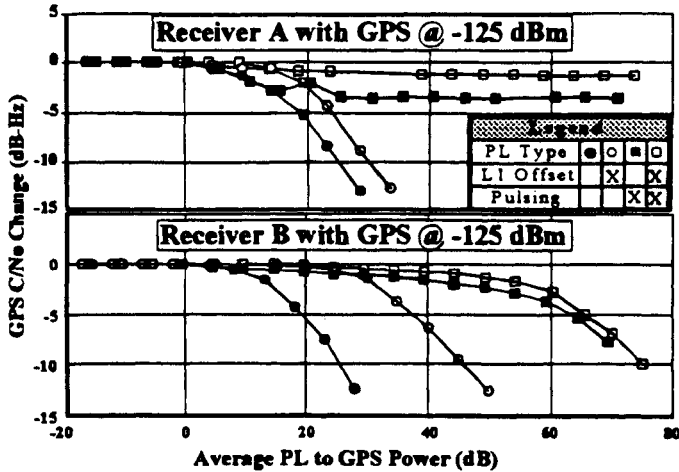


Fig. 7 GPS degradation vs pseudolite signal power with and without pulsing and frequency offset.

With no pulsing or frequency offset applied, the degradation is significant at just a 20 dB advantage in PL over GPS signal power. With either pulsing or frequency offset applied, the degradation is less, although pulsing is the more effective feature. With both applied, the degradation in reported C/N_0 from the *no PL signal* case is small (≤ 2 dB) over a range of 60 dB or more in the ratio of PL to GPS signal power. The results for this case from testing four receivers (two L_1 -C/A only and two L_1/L_2 cross-correlating types) are shown in expanded scale in Fig. 8. When compared to a user receiving a PL signal at a range of 20 n.mi. (37 km) at a level comparable to GPS, this means that a degradation of only 2 dB would be experienced by a user only 0.02 n.mi. (37 m) away but receiving the PL signal at 60 dB higher power!

Note that receiver A's degradation with a continuous signal, or with either a frequency offset or pulsing, but not both, is more than that of receiver B. This

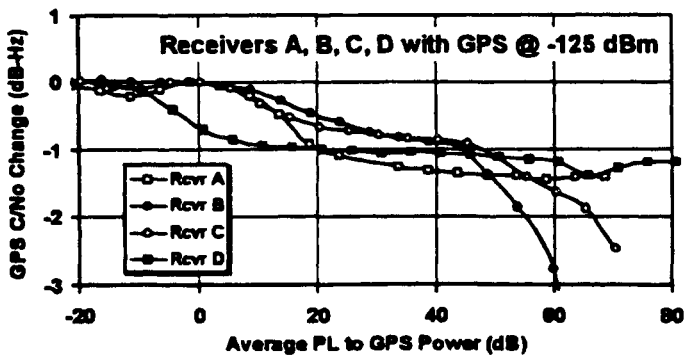


Fig. 8 GPS degradation vs pseudolite signal: 4 receivers with pulsing and frequency offset.

illustrates the difference between *soft-limiting* and *hard-limiting*. Receiver A uses multibit sampling. However, with both features turned on, receiver A's performance is quite good. Note also that receiver B's performance tails off as the PL signal gets quite strong (> 50 dB above GPS). Receiver B is a discontinued model, and receiver C is its replacement. Receiver C did not exhibit the same behavior, which was probably because of slow saturation recovery in the receiver B's front end. For receivers with fast enough saturation recovery, the results agree well with Eq. (B12) and Eq. (B13) of Appendix B.

The PL interference test results discussed above were obtained with *high-end* receivers capable of reporting signal quality data. Testing of other relatively low-cost commercial receivers was also done on a qualitative basis. These receivers were placed at varying distances from an antenna radiating a PL signal with pulsing and frequency offset features applied or not. With the PL signal off, each receiver was set up for normal GPS operations. With the PL signal on at peak power, each receiver was moved toward the PL antenna, and the separation distance was recorded when anything anomalous appeared on its display. The results for the four units tested in this manner showed no effect until within 1–5 m of the PL antenna at a peak radiated power of +15 dBm.

B. Pseudolite Data Link Testing

Tests were performed to verify the capability to transmit, receive, and demodulate the message data encoded on a PL signal with the pulsing, frequency offset, and higher data rate features described above. During laboratory tests at Stanford Telecom, a signal generator/pulser unit was interfaced directly to a GPS/PL-capable receiver. Pseudolite signals with a fixed data message were transmitted at different power levels to simulate operations at various PL/user ranges. Results of these initial tests indicated virtually error-free data reception, with the received signal quality (C/N_0) at a level of 35 dB-Hz or more.

To support planning for more comprehensive PL testing and evaluation, preliminary flight trials of a DGPS data link provided by one PL were arranged. A GPS/PL reference receiver and the signal generator/pulser unit used for the laboratory tests were installed at the FAA Technical Center (FAATC) and configured to transmit the PL signal from an antenna on the hanger roof. The test aircraft was an FAA Aerocommander (N50) with a top-mounted, low-profile GPS antenna, a GPS/PL-capable receiver, and data processing/storage equipment. For test purposes, the PL message data (LDGPS) corrections) were encoded in a 250-bit WAAS message format and transmitted without forward error correction at 250 bps.

Flight profiles flown with the test aircraft included straight-in approaches to Runway 6 at FAATC and various holding patterns ranging up to 29 km (15 n.mi.) from the PL. Received PL message data and signal quality data (C/N_0) were recorded onboard during flight segments, such as A, B, and C in Fig. 9.

Corresponding C/N_0 profiles and occurrences of individual data message errors are shown in Fig. 10. The PL data error occurrences appear to be predominantly associated with PL antenna gain reduction/obstruction phenomena during turns. Future flight trials will help address the impact of aircraft antenna type on PL signal-tracking performance. The test aircraft will be equipped to receive PL

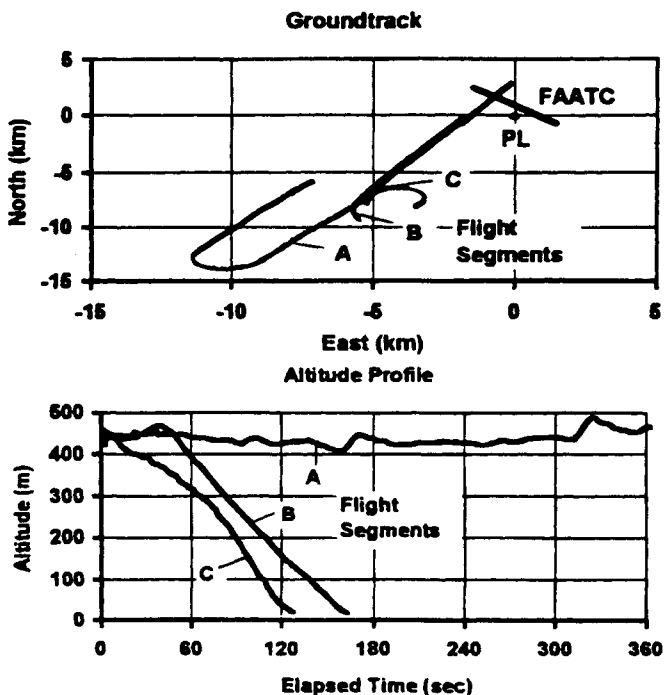


Fig. 9 Examples of flight test segments at the FAA Technical Center during pseudolite data link tests.

signals from top- and bottom-mounted, low-profile antennas, and a high-profile (blade) antenna. These trials will also assess PL data link reliability at other data rates (500 and 1000 bps).

C. Navigation Performance Testing

The initial laboratory tests and preliminary flight testing have focused on demonstrating the feasibility of PL tracking and interference mitigation to GPS signals with the pulsing and L_1 offset features applied. Additional aspects that remain to be assessed include the following:

- 1) PL code and carrier measurement quality
- 2) accuracy performance enhancement in LDGPS/PL and KDGPS/PL modes
- 3) sensitivities to PL siting (number/location) and user antenna (type/location)

Future flight trials are planned to assess navigation performance in LDGPS/PL and KDGPS/PL modes, and the sensitivity to PL siting and type/location of user antenna employed.

Independent KDGPS-based tracking of the test aircraft will be used as a truth source for assessing the consistency of PL ranging data (code and carrier) throughout a terminal area flight envelope.

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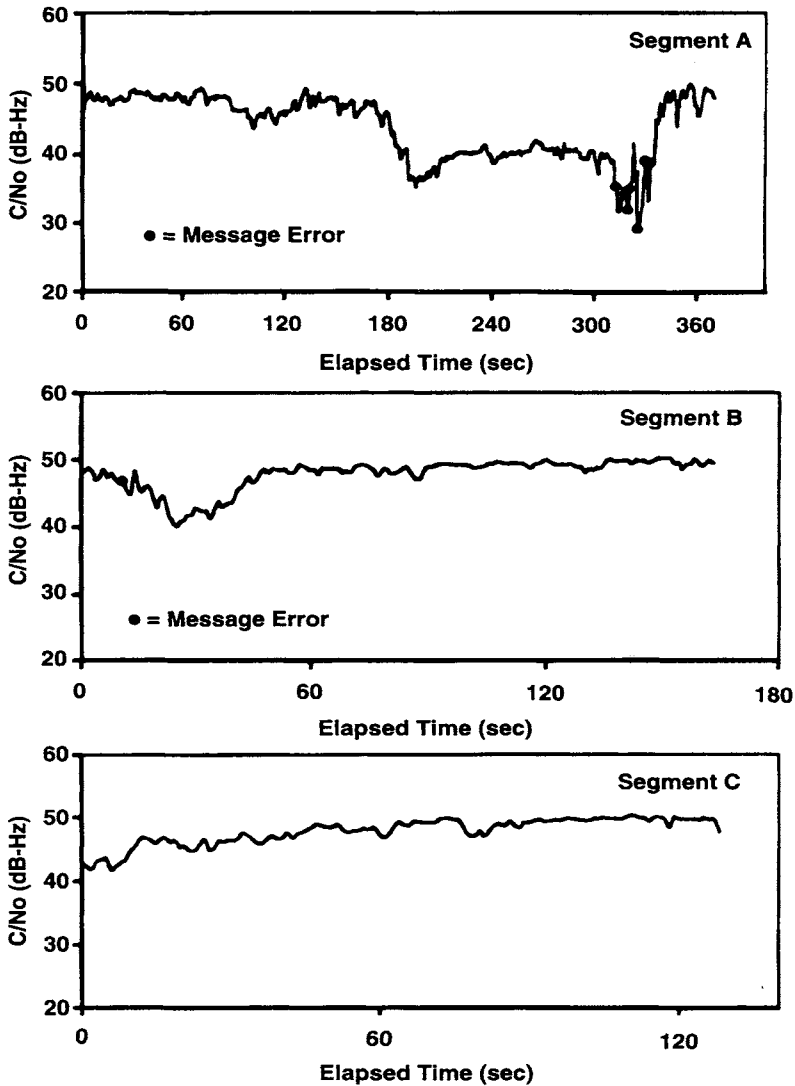


Fig. 10 Pseudolite signal profiles during data link tests.

Appendix A: Interference Caused by Cross Correlation Between C/A Codes

The effect that one C/A code has on another with respect to cross-correlation properties was described in Ref. 10 (see also Chapter 3, the companion volume). A more rigorous derivation is provided here based on the receiver signal-processing model shown in Fig. A1.

In this signal-processing model, the input signals at ①, in terms of in-phase and quadrature components, are the following desired signal:

$$\begin{aligned} I_{i1}(t) &= A_i C_i(t) \cos(2\pi \Delta f_i t + \phi_i) \\ Q_{i1}(t) &= A_i C_i(t) \sin(2\pi \Delta f_i t + \phi_i) \end{aligned} \quad (A1)$$

and the undesired signal

$$\begin{aligned} I_{j1}(t) &= A_j C_j(t + \tau_j(t)) \cos[2\pi(\delta f_j + \Delta f_j)t + \phi_j] \\ Q_{j1}(t) &= A_j C_j(t + \tau_j(t)) \sin[2\pi(\delta f_j + \Delta f_j)t + \phi_j] \end{aligned} \quad (A2)$$

where

- A_i, A_j = signal amplitudes
- $C_i(t), C_j(t)$ = signal C/A codes
- $\Delta f_i, \Delta f_j$ = signal Dopplers
- ϕ_i, ϕ_j = signal phases
- δf_j = frequency offset of undesired signal
- $\tau_j(t)$ = time offset between signals

The Doppler removal process eliminates the desired signal's Doppler and phase so that the signal components at ② in Fig. A1 are as follows:

$$\begin{aligned} I_{i2}(t) &= A_i C_i(t) \\ Q_{i2}(t) &= 0 \end{aligned} \quad (A3)$$

$$\begin{aligned} I_{j2}(t) &= A_j C_j(t + \tau_j(t)) \cos[2\pi(\delta f_j + \Delta f_j - \Delta f_i)t + \phi_j - \phi_i] \\ Q_{j2}(t) &= A_j C_j(t + \tau_j(t)) \sin[2\pi(\delta f_j + \Delta f_j - \Delta f_i)t + \phi_j - \phi_i] \end{aligned} \quad (A4)$$

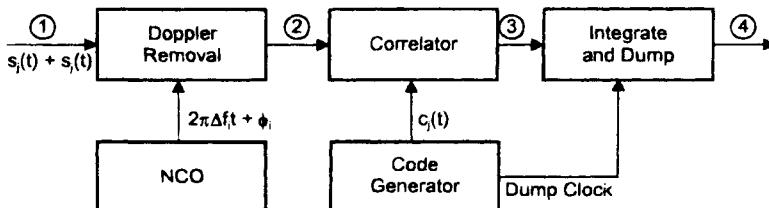


Fig. A1 Receiver signal-processing model.

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where the Doppler difference is as follows:

$$\Delta f_{ij} = \delta f_j + \Delta f_j - \Delta f_i \quad (\text{A5})$$

For simplicity, assume that the code transitions line up. Thus, for the moment, assume the following:

$$\tau_j(t) = NT_c \quad (\text{A6})$$

where N is an integer, and T_c is a chip width (1/1,023,000 s). Also, assume full correlation for signal i . Then, at ③ we have the following:

$$I_{i3}(t) = A_i \quad (\text{A7})$$

$$Q_{i3}(t) = 0$$

$$I_{j3}(t) = A_j C_j(t + NT_c) C_i(t) \cos(2\pi \Delta f_{ij} t + \phi_j - \phi_i) \quad (\text{A8})$$

$$Q_{j3}(t) = A_j C_j(t + NT_c) C_i(t) \sin(2\pi \Delta f_{ij} t + \phi_j - \phi_i)$$

Now, because of the cycle-and-add property of the C/A-codes, Eq. (A8) becomes the following:

$$I_{j3}(t) = A_j C_k(t) \cos(2\pi \Delta f_{ij} t + \phi_j - \phi_i) \quad (\text{A9})$$

$$Q_{j3}(t) = A_j C_k(t) \sin(2\pi \Delta f_{ij} t + \phi_j - \phi_i)$$

where $C_k(t)$ is another code in the same family, which is different for each value of N . The signal components at ④ are then as follows:

$$I_{i4}(t) = A_i T \quad (\text{A10})$$

$$Q_{i4}(t) = 0$$

$$I_{j4}(t) = A_j \int_0^T C_k(t) \cos(2\pi \Delta f_{ij} t + \phi_j - \phi_i) dt \quad (\text{A11})$$

$$Q_{j4}(t) = A_j \int_0^T C_k(t) \sin(2\pi \Delta f_{ij} t + \phi_j - \phi_i) dt$$

where T is a multiple M of 1 ms C/A-code repetition periods.

Power in the two correlated signals is given as follows:

$$2P_{i4} = I_{i4}^2 + Q_{i4}^2 = A_i^2 T^2 \quad (\text{A12})$$

$$2P_j(\Delta_{ij}) = I_{j4}^2 + Q_{j4}^2$$

$$= A_j^2 \left\{ \left[\int_0^T C_k(t) \cos(2\pi f_{ij} t) dt \right]^2 + \left[\int_0^T C_k(t) \sin(2\pi f_{ij} t) dt \right]^2 \right\} \quad (\text{A13})$$

Note that, through expansion using trigonometric identities, the dependence upon the phase difference has been removed in Eq. (A13), which resembles the expression for a Fourier power spectrum component at the Doppler difference. The code repeats at a 1-kHz rate, and the integration over each 1 ms code period is

identical for Doppler differences of multiples of 1 kHz. Thus, the ratio of Eq. (A13) to Eq. (A12) the cross-correlation power ratio, can be reduced to the following:

$$P_{ij}(n) = \frac{A_j^2}{A_i^2} \left\{ \left[\frac{1}{2L} \int_0^{2L} C_k \cos\left(\frac{n\pi}{L} t\right) dt \right]^2 + \left[\frac{1}{2L} \int_0^{2L} C_k \sin\left(\frac{n\pi}{L} t\right) dt \right]^2 \right\} \quad (\text{A14})$$

where

$$\Delta f_{ij} = n/2L = 1000 \text{ } n \text{ Hz}$$

$$2L = 0.001 \text{ s} = T/M$$

The term in the brackets of Eq. (A14) can be recognized as the n th power spectrum component. Obviously, Eq. (A13) can take on values for Doppler differences other than multiples of 1 kHz, depending upon the value of T , but peaks at the 1 kHz lines. That is, there is cross-correlation at other Doppler differences, but less than at the 1 kHz lines. Because of the navigation message databits, M is limited to 20. If T were allowed to be infinite (very large), the cross correlation at these other values would approach zero. For example, if T was a large multiple of Doppler difference cycle periods, integration over each cycle would pick up a different portion of the code, or multiple codes plus a different portion of the code. If we take a Doppler difference of 50 Hz and integrate over 20 ms, then integration over the last 10 ms would cancel that over the first 10 ms, because the codes would simply be flipped. This is true for any Doppler difference that is a submultiple of 1 kHz and an integration time that is an integer number of Doppler difference cycle periods. For other Doppler differences, other than the multiples of 1 kHz, a longer integration period is required for eventual cancellation.

Equation (A14) was evaluated for equal amplitude signals for all relative code phases as defined in Eq. (A6) for two specific GPS C/A-code pairs. The first pair (PRN6/PRN28) is considered to be the *worst* pair of GPS codes; whereas the second pair (PRN7/PRN201) is made up of the *best* GPS code and the *best* selected for Inmarsat-3.⁹ For each pair, the maximum spectrum components over all possible integer code phases ($0 \leq N \leq 1022$), are plotted in Fig. A2 for Doppler differences up to 20 kHz. The maximum spectrum components are also plotted in Fig. A3, for Doppler differences in the range 1017–1029 kHz, which corresponds to one of the signals transmitting in the first null of the other. This ± 6 kHz Doppler range represents the maximum expected from satellite and user motion. The average over all N computed for the PRN6/PRN28 pair is also plotted in Fig. A3 for comparison. The average is approximately equal to the following spectral line envelope:

$$S(f) = \frac{1}{1023} \frac{\sin^2(n\pi/1023)}{(n\pi/1023)^2} \quad (\text{A15})$$

which is about 8–9 dB below the maximum values. Note that in both figures, it doesn't seem to matter which code pair is used when determining the worst-case

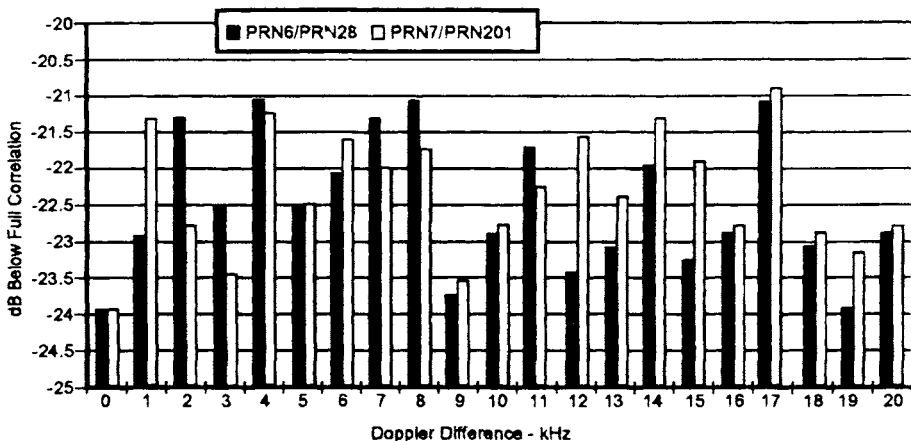


Fig. A2 Maximum cross-correlation spectral components for two code pairs in Doppler difference range of 0–20 kHz.

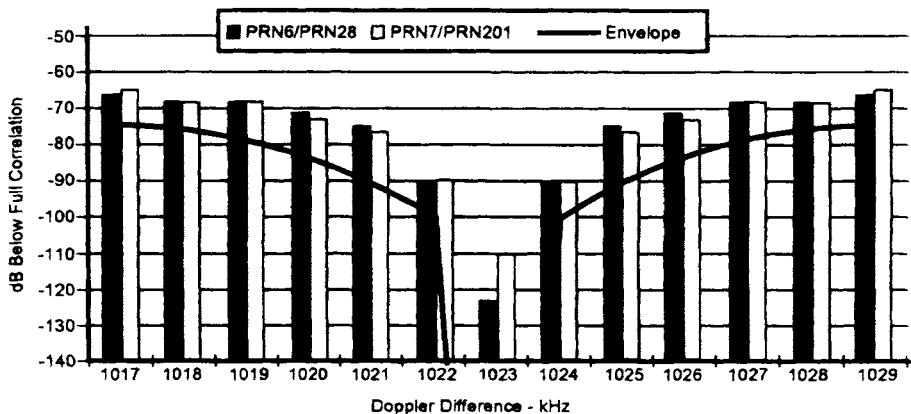


Fig. A3 Maximum cross-correlation spectral components for two code pairs in Doppler difference range of 1017–1029 kHz.

magnitudes. This is because each of the 1023 code phases ($0 \leq N \leq 1022$) results in a different code. Consequently, codes with bad (i.e., large) spectral components are generated in each case, and some of these codes are unbalanced, as well.

Although some of the *worst*-case components shown in Fig. A3 slightly exceed the predicted -80 dB level stated in Sec. IIB1, they are still typically below -70 dB. Within ± 2 kHz of the null, they are, in fact, below -80 dB. More significantly, all components are a good 50–60 dB lower than the level near center frequency. This is in addition to the margin realized from pulsing with a $1/11$ duty cycle. (See Appendix B)

The derivation presented above applies when the cross-correlation code transitions are lined-up, which, of course, they rarely will be. Spilker¹⁰ points out that,

at the n kHz carrier frequency differences, there also exists a code frequency shift, which is less by a factor of 1540. Thus, for example, at the 1 kHz carrier frequency difference, there is a code frequency difference of $1000/1540 \approx 0.65$ chips per second. Thus, the code transitions will not stay lined-up. This is especially true for a large frequency difference of 1.023 MHz, in which case the code frequency difference is approximately 664 chips per second. McGraw¹² pointed out that cross-correlation levels can be even higher when the code transitions are not lined up. However, because of the rapidly changing time relationships between the codes, the resulting cross-correlation becomes noise-like, and simply becomes a noise interference. The effect of this interference is addressed in Appendix B.

Appendix B: Interference Caused by Pseudolite Signal Level

One C/A-code signal can interfere with another if it is strong enough, independent of the cross-correlation. This can certainly happen in the case of a PL signal, which may interfere with satellite signals as well as with other PL signals. In other words, a PL signal is a noise source that may jam the other signals unless measures are taken to mitigate this effect. The following provides an assessment of the impact of that jamming and a method for minimizing it; namely, by pulsing the PL signal.

Background

In general, the spreading process in a receiver's correlator is defined as a signal (noise or otherwise) being passed through a filter described with a frequency response equal to the spectral density of the PRN code. That is, the interference noise density at the output of the correlator is as follows:

$$N_{0f} = \int_{-\infty}^{\infty} S_c(f) S_f(f) df \quad (B1)$$

where $S_c(f)$ is the spectral density of the reference PRN code and $S_f(f)$ is the density of the interference or noise. The reference C/A-code has a discrete spectral density that can be described as follows:

$$S_c(f) = \sum_{n=-\infty}^{\infty} c_n \delta(f - 1000 n) \quad (B2)$$

where the c_n are spectral line coefficients; $\delta(f)$ is the dirac delta function; and the c_n vary about the envelope of Eq. (A15) in Appendix A. The reference C/A-code spectral density has the property that

$$\int_{-\infty}^{\infty} S_c(f) df = 1 \quad (B3)$$

First consider bandlimited thermal noise with density N_0 with a two-sided intermediate frequency (IF) bandwidth of B_i (brick wall filter).

$$N_{0T} = N_0 \int_{-B_i/2}^{B_i/2} S_c(f) df \leq N_0 \quad (B4)$$

The variation of the C/A-code spectral lines averages out over the wide bandwidth.

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A similar equation applies for wide- and narrow-band interference with a spectral density as follows:

$$S_i(f) = \frac{P_i}{f_u - f_l} \quad (B5)$$

for upper and lower frequency limits f_u and f_l (converted to IF) and total interference power P_i (relative to the signal power S).

$$N_{0I} = \frac{P_i}{f_u - f_l} \int_{\max(-\alpha_1, \alpha_3)}^{\min(\alpha_1, \alpha_2)} S_c(f) df \quad (B6)$$

where

$$\alpha_1 = B_I/2, \alpha_2 = f_u - f_{IF} \quad \text{and} \quad \alpha_3 = f_l - f_{IF}.$$

For narrow bandwidth noise interference centered at the GPS frequencies (i.e., somewhat less than the code chipping rate $1/T_c$), this equation becomes as follows:

$$N_{0I} \approx P_i T_c \quad (B7)$$

This is only true for the C/A code where interference bandwidths are on the order of 100 kHz or greater because of the variations in the line spectrum of the codes.

Pseudolite Interference

Now assume that the interference is another C/A-code signal with the following spectral density:

$$S_{PL}(f) = P_{PL} \sum_{n=-N_B}^{N_B} c'_n \delta(f - 1000 n \pm 1.023 \times 10^6) \quad (B8)$$

where P_{PL} is the received PL signal power, and N_B indicates the band-limiting effect. Because the summations in Eqs. (B2) and (B8) are over a large number of spectral components, it suffices to use the average envelope of Eq. (A15), divided by 1000 to spread the components into a continuous spectral density, for evaluation. Then, Eq. (B1) can be approximated with the following integral:

$$N_{0PL} \approx P_{PL} T_c^2 \int_{-B_I/2}^{B_I/2} \frac{\sin^2(\pi f T_c) \sin^2[\pi(f \pm 1/T_c) T_c]}{(\pi f T_c)^2 [\pi(f \pm 1/T_c) T_c]^2} df \quad (B9)$$

This can be compared to the interference noise density from one GPS satellite j to another satellite i at the normal frequency given by the following:

$$N_{0S_{ij}} \approx P_{S_j} T_c^2 \int_{-B_I/2}^{B_I/2} \frac{\sin^4(\pi f T_c)}{(\pi f T_c)^4} df \quad (B10)$$

In general, Eqs. (B9) and (B10) must be evaluated numerically. This was done over a wide bandwidth of $\pm 10/T_c$, resulting in the following relationship:

$$10 \log_{10} \left(\frac{N_{0PL}}{N_{0S_{ij}}} \right) = 10 \log_{10} \left(\frac{P_{PL}}{P_{S_j}} \right) - 8.19 \text{ dB} \quad (B11)$$

which indicates that the PL interferes by 8.19 dB less by transmitting in the null, than it would if it were transmitting at the same frequency as the satellites. However, this is not enough!

The first term of Eq. (B11) can become significant as the user receiver comes closer to a PL. This is a key reason for PL pulse modulation, which has the effect of only interfering a percentage of the time equal to the pulse duty cycle, provided that the receiver clips the pulses. This results in a loss in received satellite C/N_0 of either:

$$L = -10 \log_{10} \left[1 - DC + \frac{2R_{\Delta f}}{3} \left(\frac{L_{\max}}{L_N} \right)^2 \frac{DC}{(1 - DC)^2} B_f T_c \right] \quad (\text{B12})$$

or

$$L = 10 \log_{10} \left[1 + \frac{2R_{\Delta f}}{3} \frac{P_{PL}}{N_0} T_c \cdot DC \right] \quad (\text{B13})$$

depending upon whether or not the PL signal is saturating the receiver's analog-to-digital (A/D) sampler (*soft* or *hard-limiting*), where $R_{\Delta f}$ is the reduction in interference realized using a frequency offset (0.1517, if ± 1.023 MHz, and 1, if on frequency), $L_{\max}$ is the maximum sampler threshold level, L_N is the one sigma noise level in terms of the threshold level and P_{PL}/N_0 is the average received PL signal-to-noise density. Eq. (B12) represents the loss when the PL pulses are saturating the A/D and Eq. (B13) represents the loss when they are not. Note that in the former case, the loss is proportional to the ratio of IF bandwidth to the code bandwidth.

Cross-correlation can still occur, even if the pulses are clipped. However, as was shown in Appendix A, the cross-correlation is reduced substantially by PL transmission in the first null of the satellite C/A-code/spectrum ($L_1 \pm 1.023$ MHz). Pseudolite transmission at higher nulls is also possible, but this begins to add more complexity to a receiver designed to process both GPS and PL signals. Another key reason for pulsing is to prevent capturing the front end of the user's receiver. This is especially important for hard-limiting receivers (1-bit samplers) that normally do not employ front-end AGC circuits.

Appendix C: Navigation Filter Modeling with Pseudolite Measurements

A GPS pseudorange measurement (and carrier range) is generally a nonlinear function of the satellite position vector for ($\mathbf{r}_s$) and the user position vector ($\mathbf{r}_u$). This measurement is modeled by the following:

$$z = g + b_u + v \quad (\text{C1})$$

where

$$g = |\mathbf{r}_s - \mathbf{r}_u| \quad (\text{C2})$$

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is the geometric range; b_u is the user clock offset from GPS time, and v represents the composite of uncorrected measurement errors caused by atmospheric, satellite timing offsets, multipath, and noise. Given the satellite ephemeris and an a priori estimate of the user location ($\mathbf{r}_u$), then g can be represented by the Taylor series expansion:

$$g = g_0 + \mathbf{h}\Delta\mathbf{r}_u + \Delta\mathbf{r}_u^T J \Delta\mathbf{r}_u / 2 + \dots \quad (\text{C3})$$

where

$$\Delta\mathbf{r}_u = \mathbf{r}_u - \mathbf{r}_u^- \quad (\text{C4})$$

$$g_0 = |\mathbf{r}_s - \mathbf{r}_u^-| \quad (\text{C5})$$

$$\mathbf{h} = \partial g / \partial \mathbf{r}_u |_{\mathbf{r}_u = \mathbf{r}_u^-} \quad (\text{C6})$$

$$J = \partial^2 g / \partial \mathbf{r}_u \partial \mathbf{r}_u |_{\mathbf{r}_u = \mathbf{r}_u^-} \quad (\text{C7})$$

Linear Measurement Model

Given the large user/satellite separation and slowly changing geometry, common practice is to employ an extended (linearized) Kalman filter that encompasses only the first two terms for g in Eq. (C3). The standard EKF equations for updating the a priori estimate of user position ($\mathbf{r}_u$) and clock bias (b_u) are as follows:

$$\begin{aligned} \mathbf{x}_u^+ &= \mathbf{x}_u^- + \mathbf{k}(z - g_0) \\ \mathbf{k} &= \mathbf{P}^- \mathbf{h}^T / (\mathbf{h} \mathbf{P}^- \mathbf{h}^T + \sigma_v^2) \\ \mathbf{P}^+ &= (\mathbf{I} - \mathbf{k} \mathbf{h}) \mathbf{P}^- (\mathbf{I} - \mathbf{k} \mathbf{h})^T + \mathbf{k} (\sigma_v^2) \mathbf{k}^T \end{aligned} \quad (\text{C8})$$

where

$$\mathbf{x}_u \equiv \begin{pmatrix} \mathbf{r}_u \\ b_u \end{pmatrix} \quad (\text{C9})$$

and

$$\mathbf{h} \equiv [(\mathbf{r}_s - \mathbf{r}_u)^T / r_s \quad 1] \quad (\text{C10})$$

Also, $\mathbf{P}^-$ is the covariance of the a priori state estimate ($\mathbf{x}_u$), and σ_v^2 is the variance of the measurement error v , assumed to be Gaussian white noise.

Nonlinear Measurement Model

When PL measurements are introduced, the user/PL range is much less and more dynamic. Consequently, a Gaussian second-order filter¹⁹ can be employed that accounts for the measurement nonlinearity with the quadratic component included

$$z - g_0 = \mathbf{h}\Delta\mathbf{x}_u + \Delta\mathbf{r}_u^T J \Delta\mathbf{r}_u / 2 + v \quad (\text{C11})$$

The corresponding GSO filter equations are expressed by the following:²⁰

$$\begin{aligned} \mathbf{x}_0^+ &= \mathbf{x}_u^- + \mathbf{k}(z - g_0 - \eta) \\ \mathbf{k} &= \mathbf{P}^- \mathbf{h}^T / (\mathbf{h} \mathbf{P}^- \mathbf{h}^T + \sigma_v^2 + \sigma_\eta^2) \\ \mathbf{P}^+ &= (\mathbf{I} - \mathbf{k} \mathbf{h}) \mathbf{P}^- (\mathbf{I} - \mathbf{k} \mathbf{h})^T + \mathbf{k} (\sigma_v^2 + \sigma_\eta^2) \mathbf{k}^T \\ \eta &= \text{Tr}[\mathbf{J} \mathbf{P}^-] / 2 \\ \sigma_\eta^2 &= \text{Tr}[\mathbf{J} \mathbf{P}^- \mathbf{J} \mathbf{P}^-] / 2 \end{aligned} \quad (\text{C12})$$

where $\mathbf{x}_u$, $\mathbf{h}$, and $\mathbf{J}$ are as defined in this appendix. The new components account for the bias and measurement variance introduced by the quadratic nonlinearity.

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